Case A: Neglect the dynamics of the motor (La=0) and consider the motor shaft as rigid (kt→∞).

In this case the equation of the system becomes

$$\begin{cases} (J_m + (mt + mp)\tau^2)\ddot{\theta}_m + C_d\tau^2\dot{\theta}_m = C_m = k_\psi i_a \\ R_a i_a + k_\psi \dot{\theta}_m = V_a \rightarrow \\ i_a = \dfrac{1}{R_a}V_a - \dfrac{k_\psi}{R_a}\dot{\theta}_m \end{cases}$$

So that the final equation used in the MATLAB Simulink is as follows

$$(J_m + (mt + mp)\tau^2)\ddot{\theta}_m + (C_d\tau^2 + \frac{{k_\psi}^2}{R_a})\dot{\theta}_m = \frac{k_\psi}{R_a}V_a$$

1.

The feedforward control logic is like the picture considering the motion law for input. Hence, we should use position and velocity to earn $V_{\text{a-ref}}$ to use it as the input. As shown equation below.

$$V_{a,ref} = \frac{R_a}{\tau k_\psi}[(J_m + (mt + mp)\tau^2)\ddot{x} + (C_d\tau^2 + \frac{{k_\psi}^2}{R_a})\dot{x}]$$

The result of the Simulink is shown below the blue is reference and the orange is response.

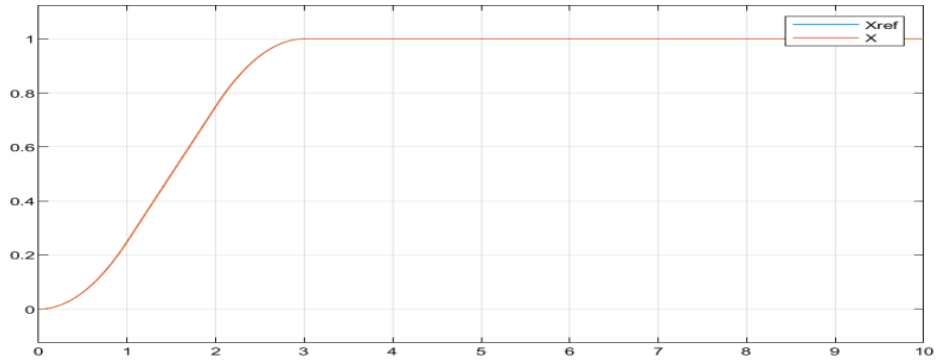

Figure 1. position FF control strategies.

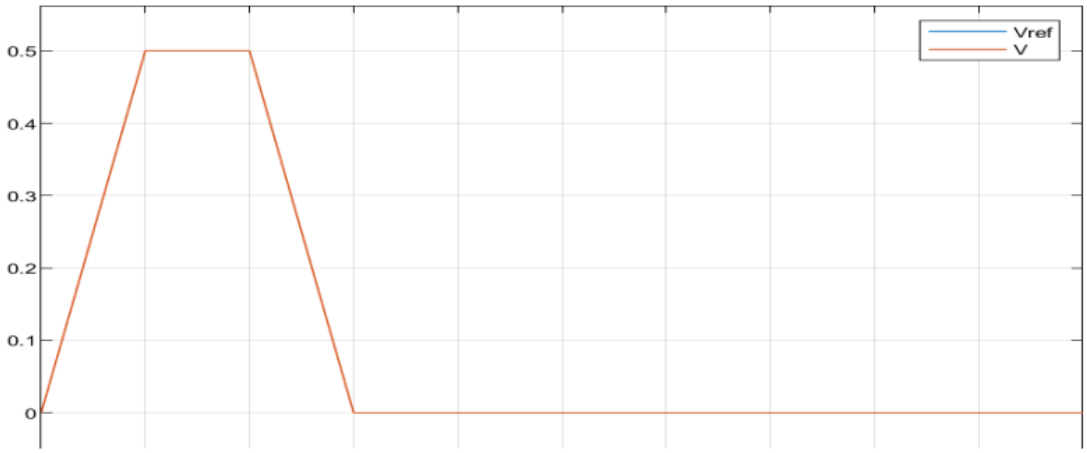

Figure 2. speed in FF control strategies.

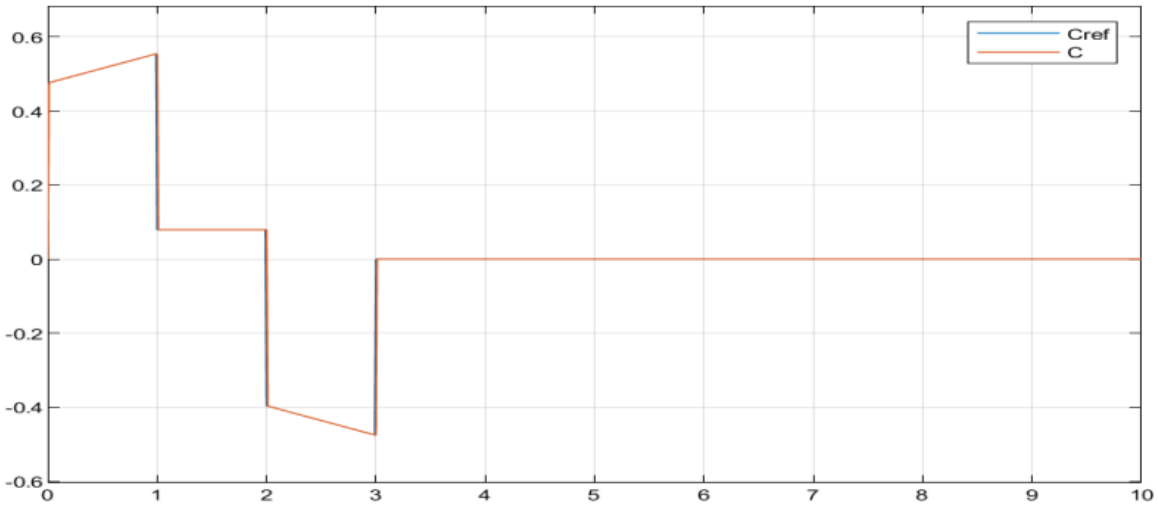

Figure 3. the torque of the motor in FF

2. **Feedback (PI controller) by turning** the result is shown below.

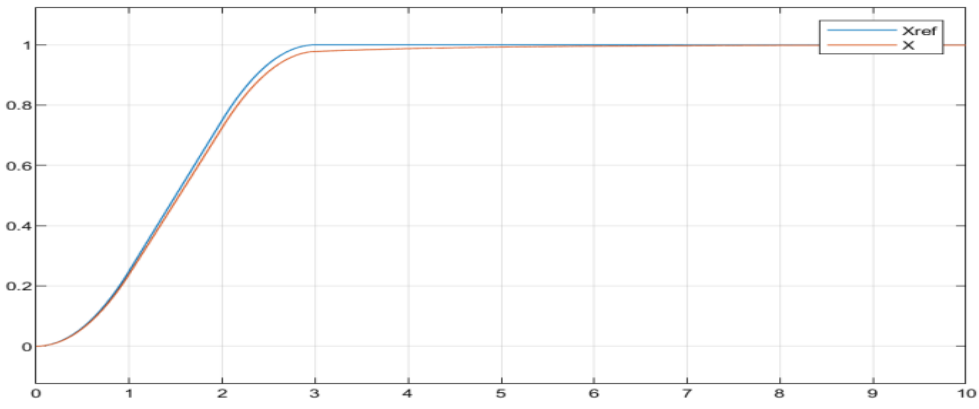

Figure 4. position in FB control logic.

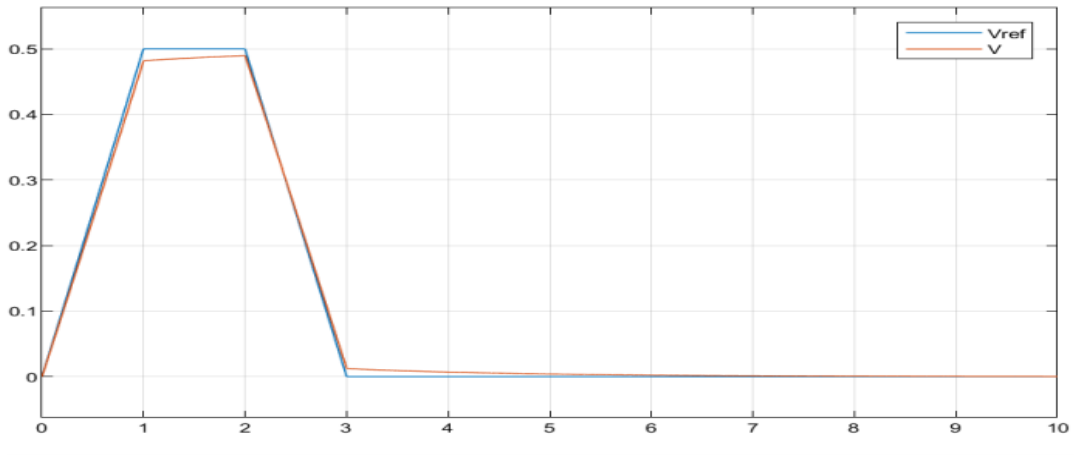

Figure 5. speed in FB control logic

3. With uncertainties.

FF with uncertainties (damping coefficient is higher than in other cases) the results plots are shown below.

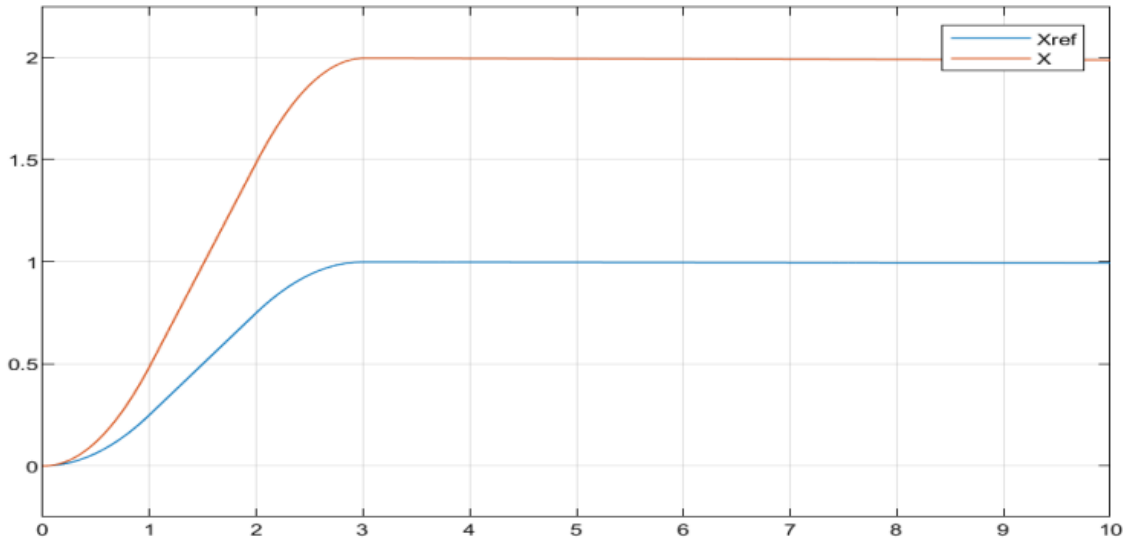

Figure 6. position of the table in FF with uncertainties

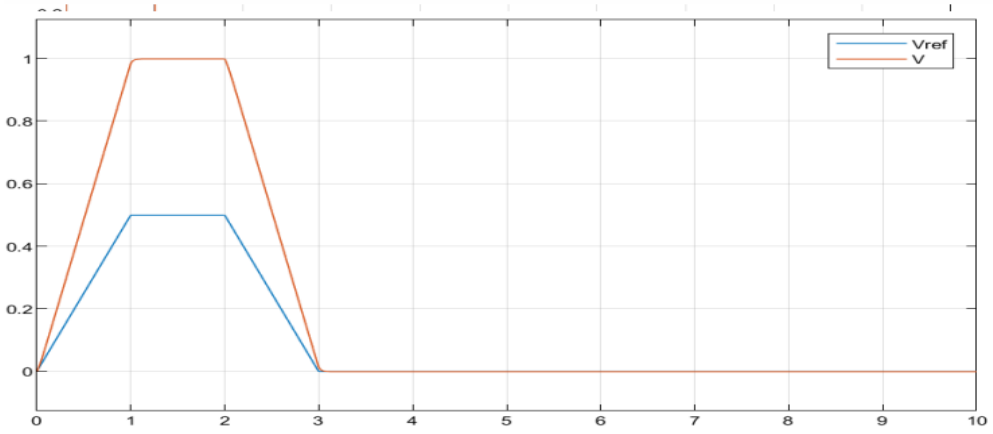

Figure 7. speed of the table in FF with uncertainties.

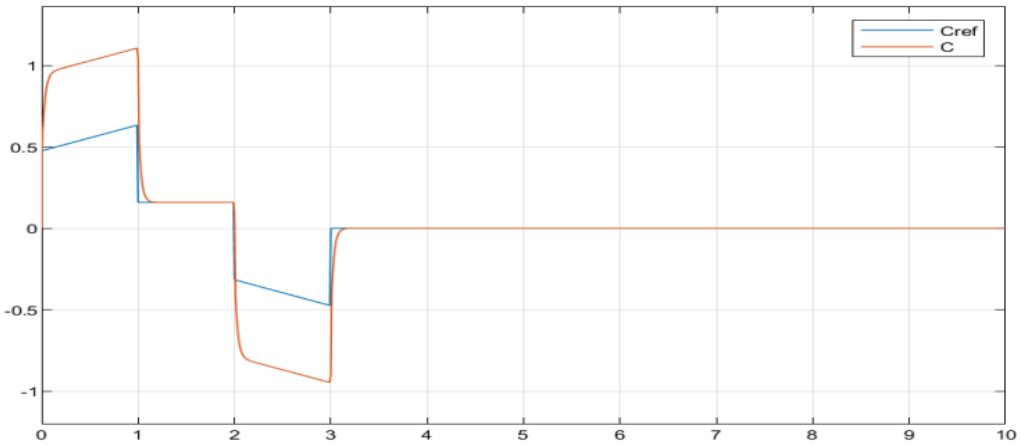

Figure 8. toque of the motor in FF with uncertainties.

4. Presence of the disturbances acting on the table.

a. Feedforward control strategy.

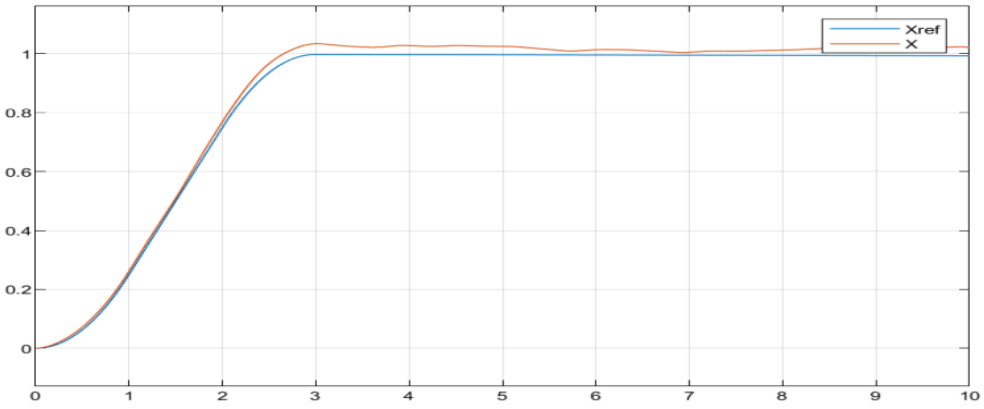

Figure 9. position of the table in FF in presence of disturbances

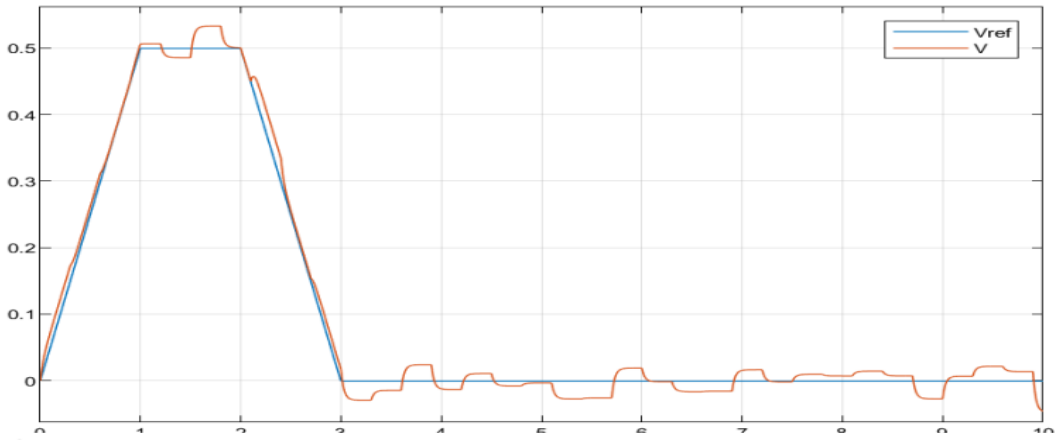

Figure 10. speed of table in FF in presence of disturbances.

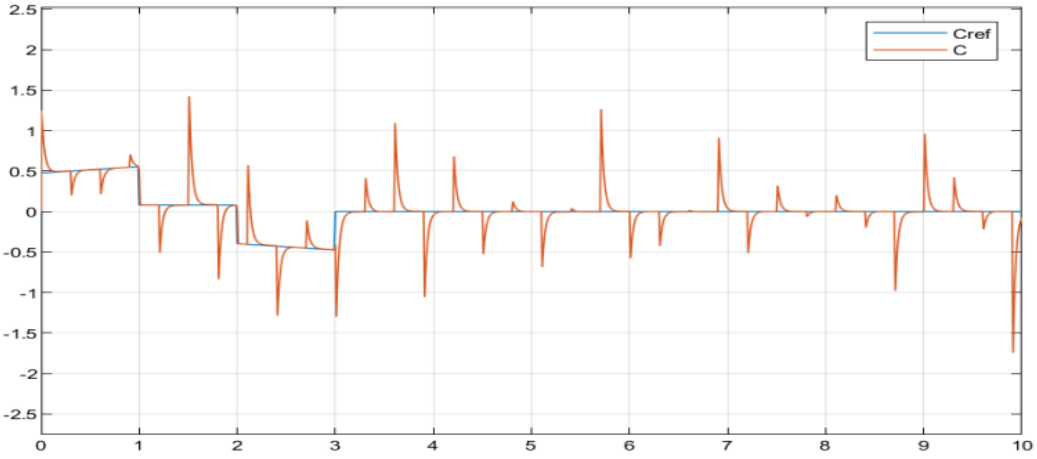

Figure 11. torque of the motor in FF in presence of disturbances.

b. Feedback control logic.

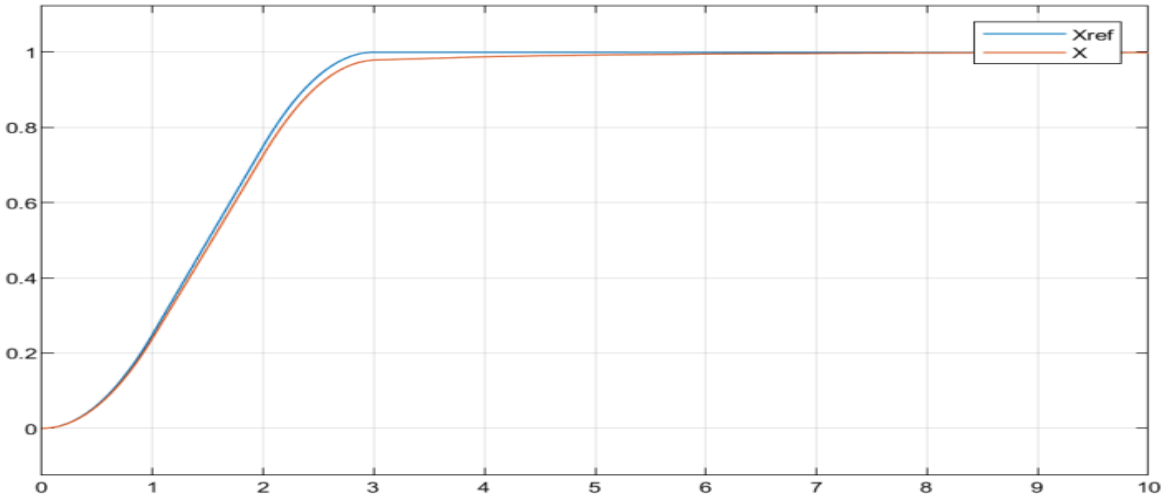

Figure 12. the position of table in FB in presence of the disturbances.

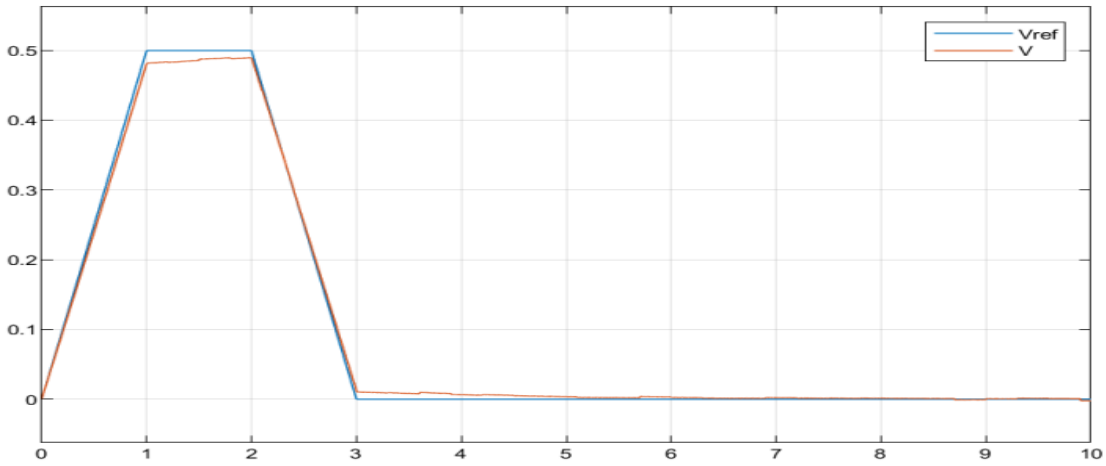

Figure 13. speed of the table in FB in presence of disturbances.

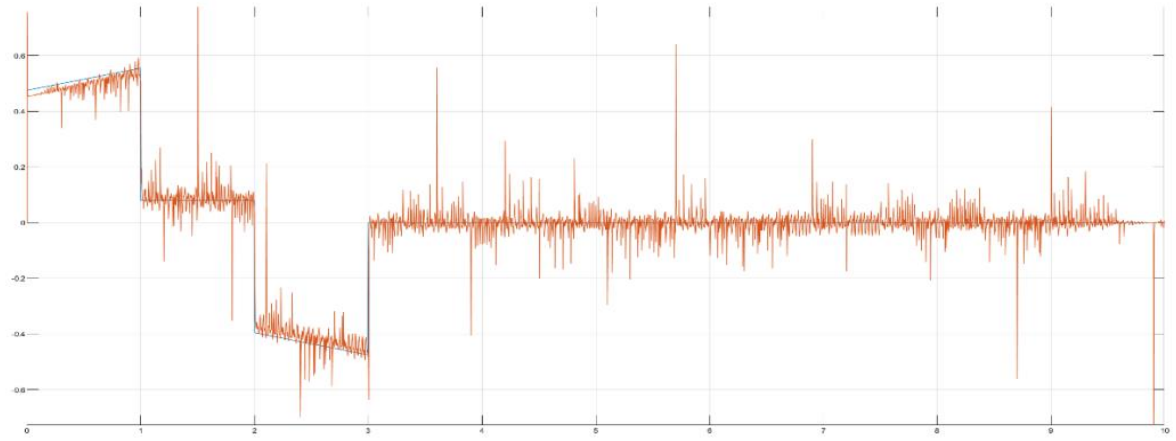

Figure 14. torque of the motor in FB in presence of disturbances.

5. Considering the presence of noise in FB control logic.

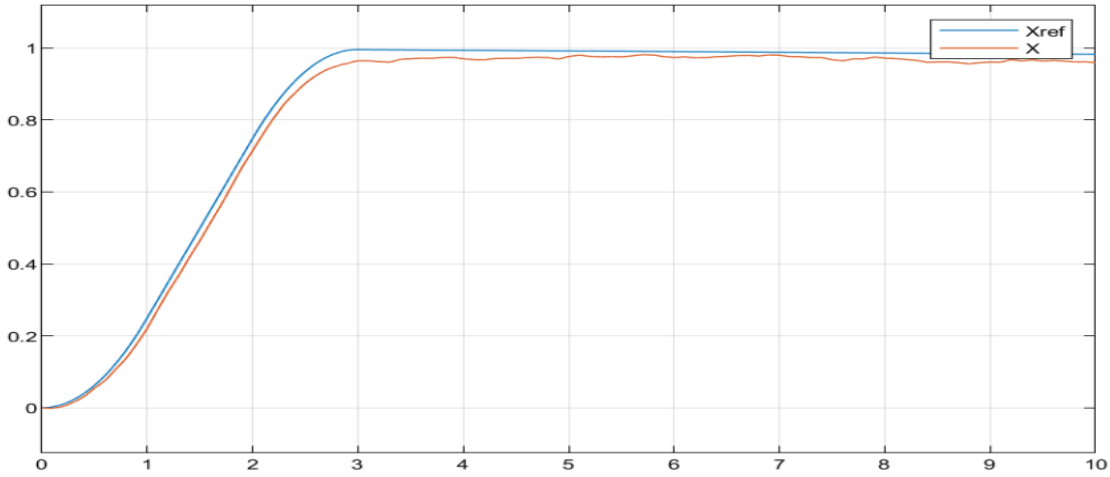

Figure 15. position of the table in FB logic in presence of noises.

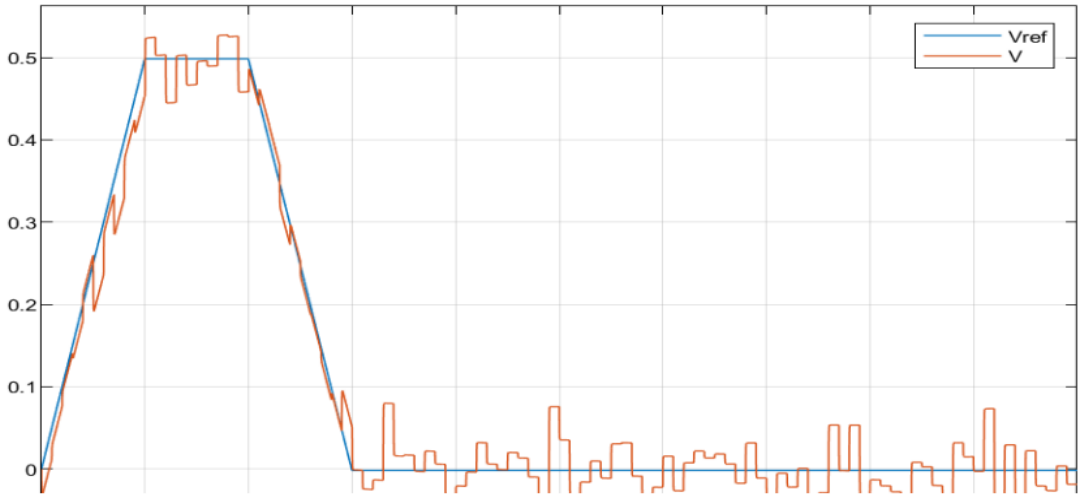

Figure 16. speed of the table in FB logic in presence of noises

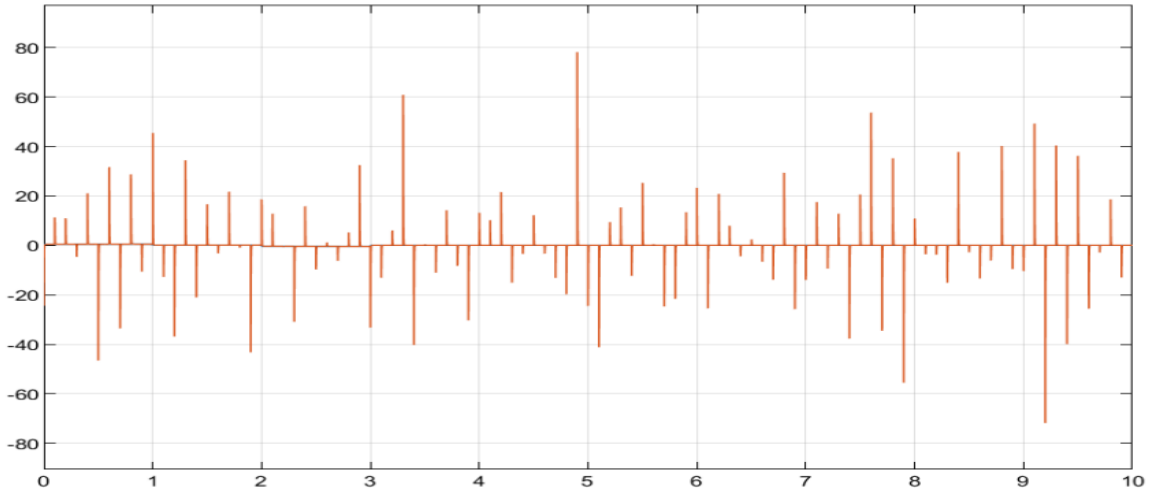

Figure 17. the torque of the motor in FB logic in presence of noises

Case B: Consider the dynamics of the motor (La≠0) and still consider the motor shaft as rigid (kt→∞).

The equation of the motion of the system become as shown below in this case.

$$Laplace\begin{cases}((J_m+(mt+mp)\tau^2)S^2+C_d\tau^2S)\theta(S)=C_m=k_\psi i_a\\(L_aS+R_a)I_a(S)=V_a-k_\psi S\theta(S)\rightarrow\\I_a(S)=\dfrac{1}{(L_aS+R_a)}V_a-\dfrac{k_\psi S}{(L_aS+R_a)}\theta(S)\end{cases}$$

1. Feedforward control logic

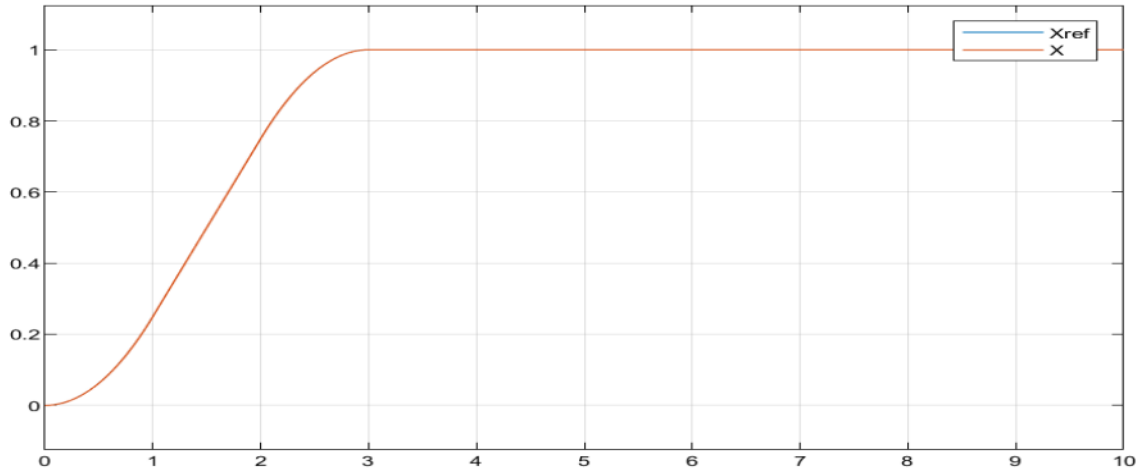

Figure 18. position of the table in FF control logic, case B.

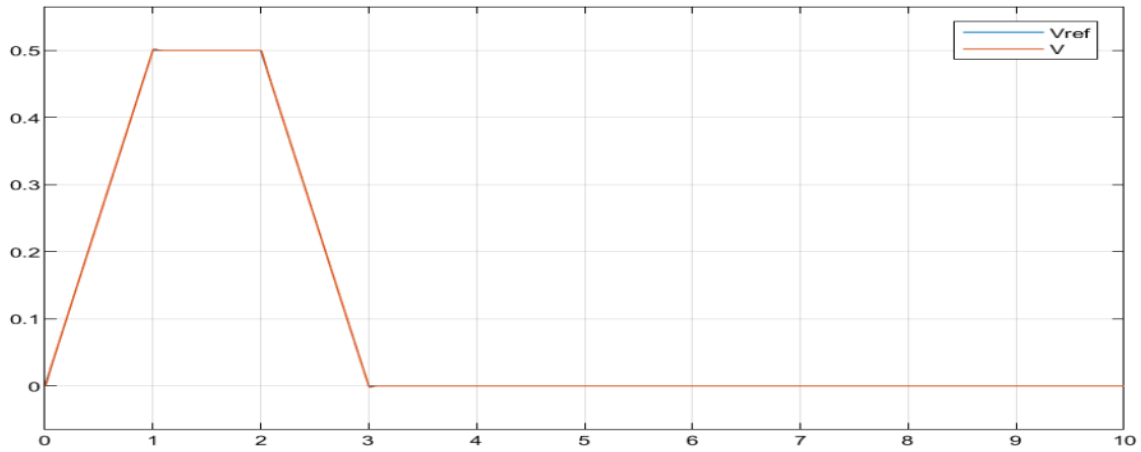

Figure 19. speed of the table, in FF control logic in case B

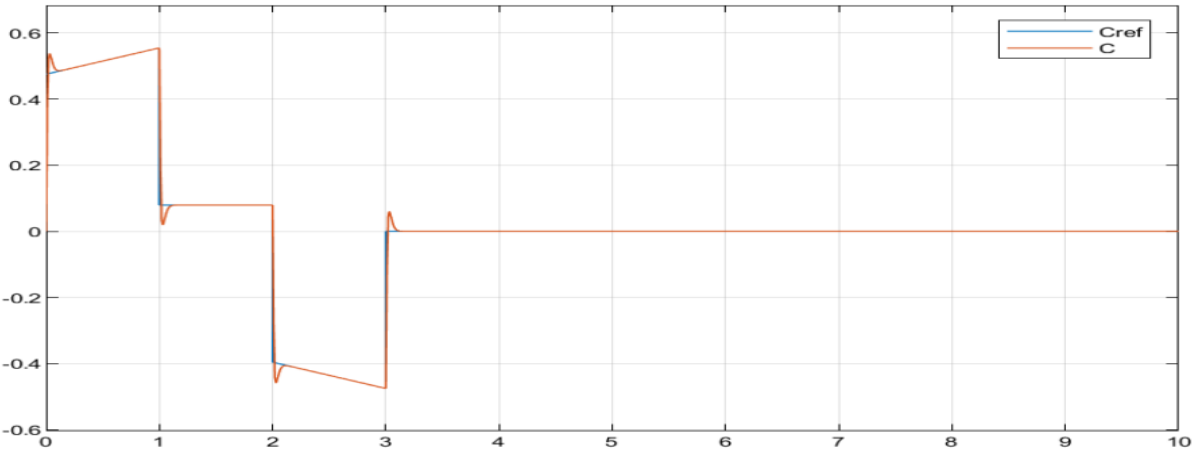

Figure 20. torque of the motor, in FF control logic in case B.

2. FB control (PI controller)

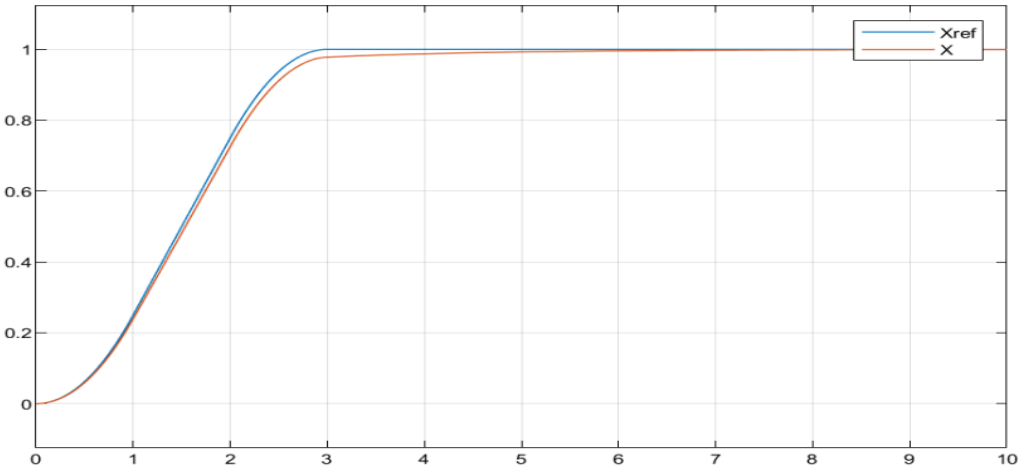

Figure 21. position of the table in FB control logic in case B.

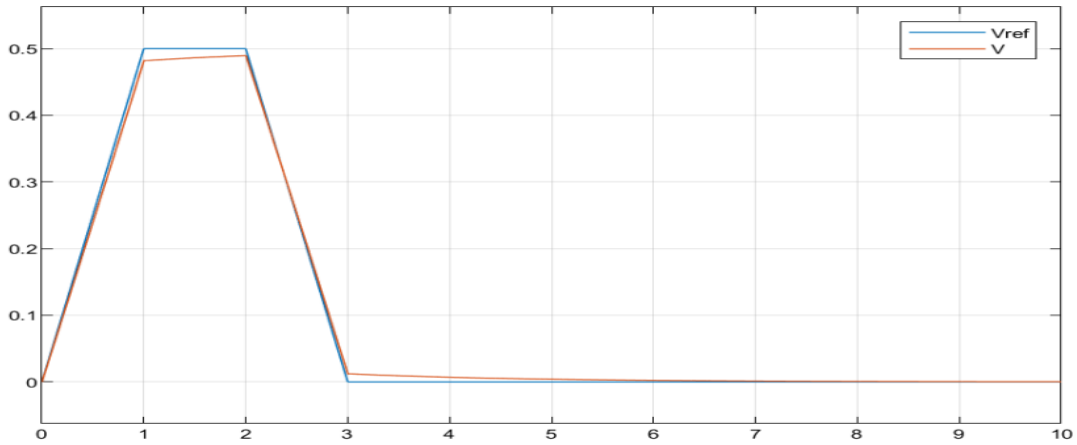

Figure 22. speed of the table in FB control logic in case B.

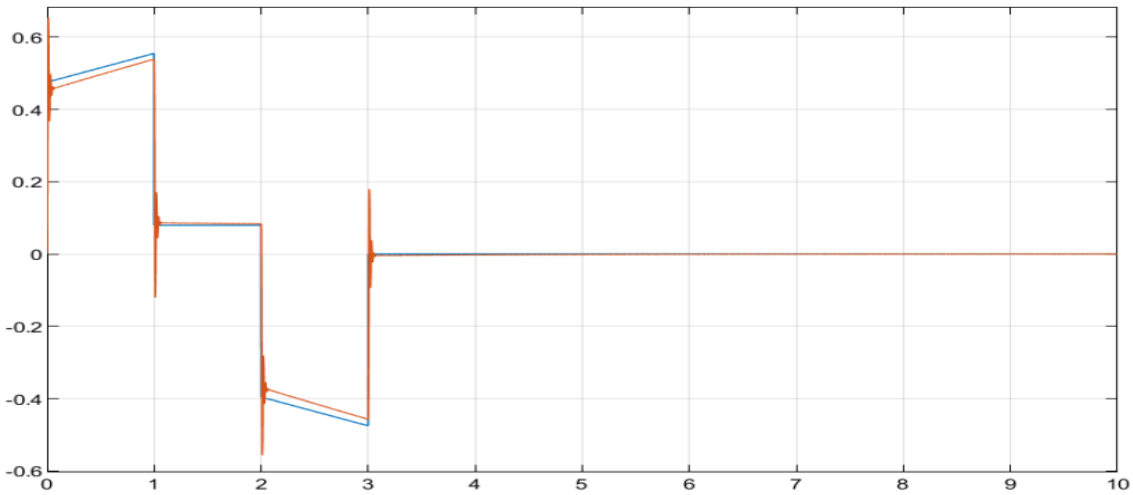

Figure 23. toque of the motor in FB control logic in case B.

Case C: Neglect the dynamics of the motor (La=0) and consider a finite torsional stiffness for the motor shaft.

1. **FB co-located control (PI controller)**

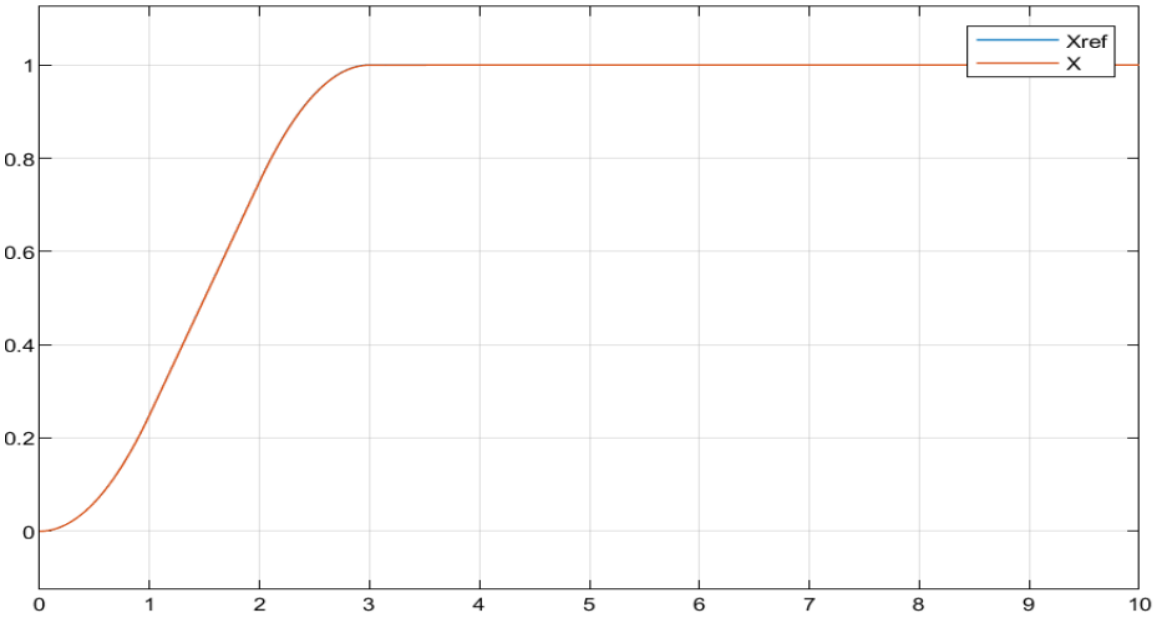

Figure 24. position of the table in FB control logic in co-located.

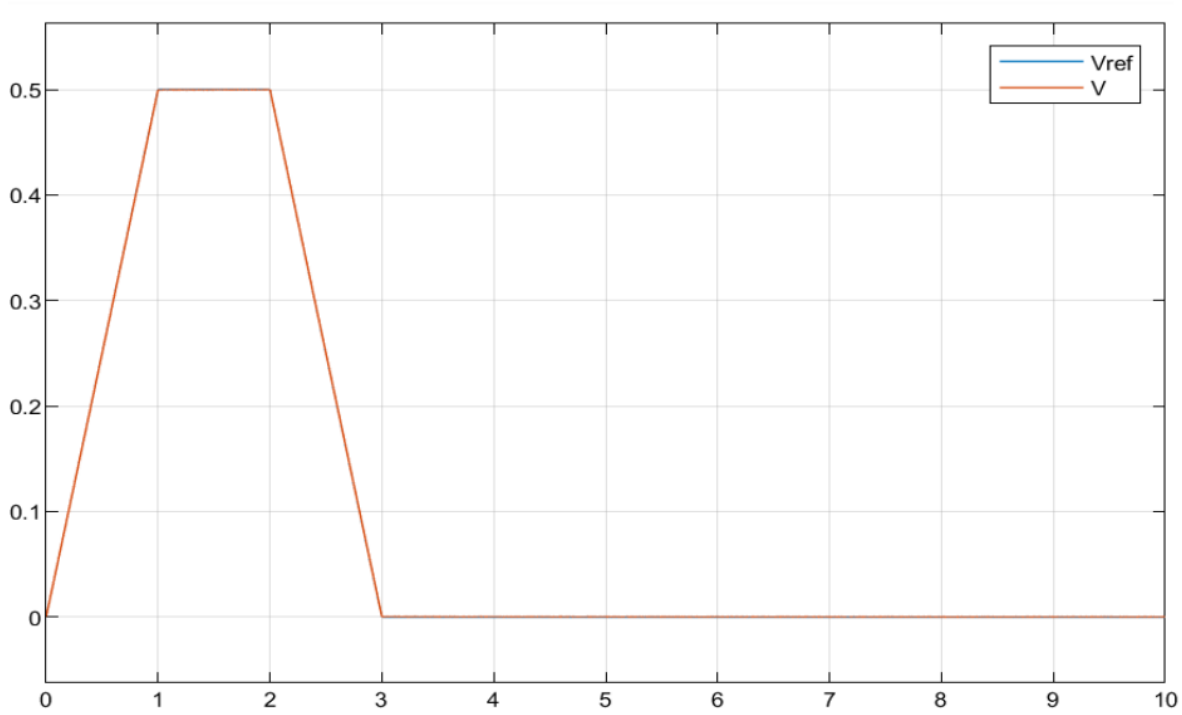

Figure 25 speed of table in FB control strategy in co-located.

2. FB non-co-located control (PI controller)

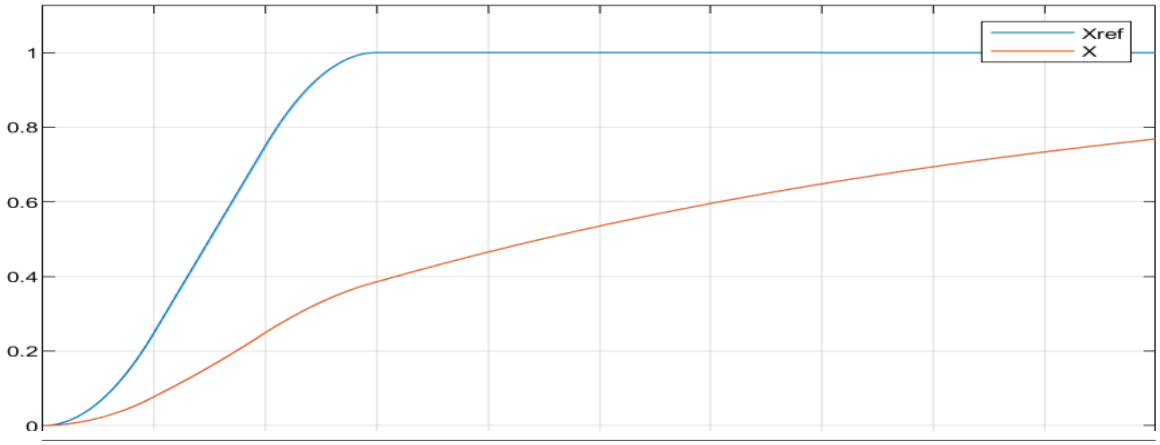

Figure 26. position of the table in FB control logic in non-co-located.

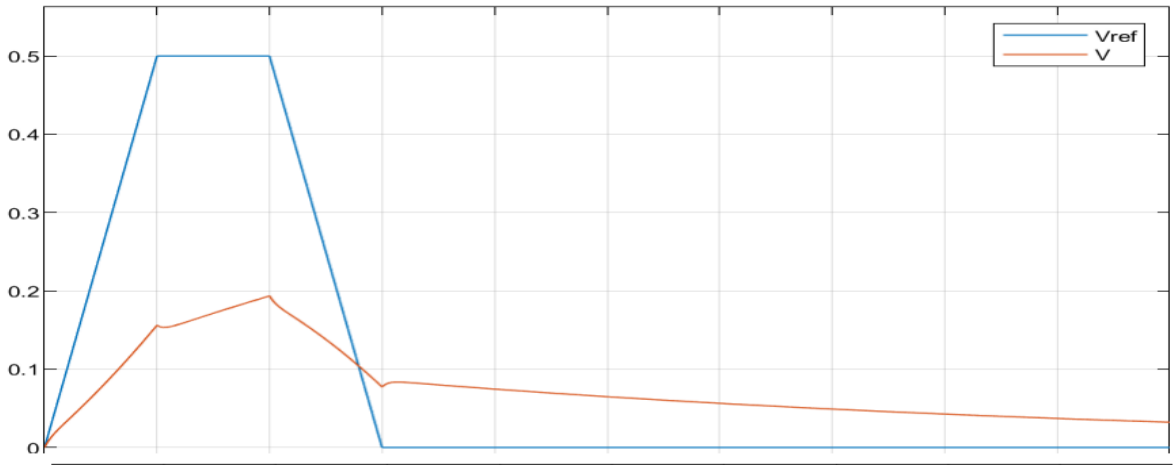

Figure 27 speed of the table in FB control logic in non-co-located.

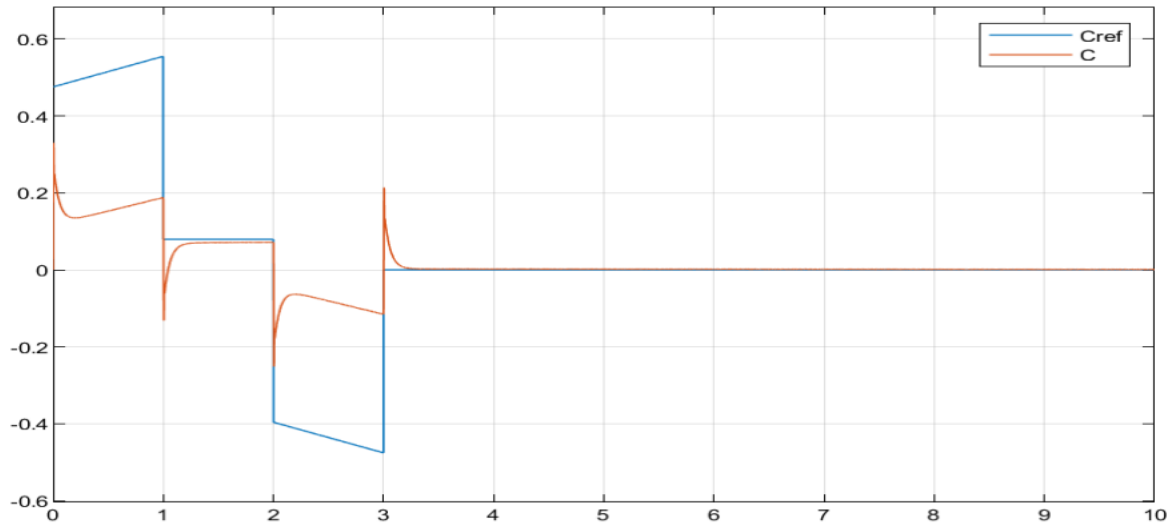

Figure 28. torque of the motor in FB control logic in non-co-located.